UNIVERSITY OF NOTRE DAME
Department of Civil and Environmental Engineering and Earth Sciences

CE 60130 Finite Elements in Engineering **April 17th, 2018**
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Project Part I

Develop a code to solve the time dependent advection equation written in conservative form:

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x}(uC) + \frac{\partial}{\partial y}(vC) = 0$$

on a square domain $\Omega = [-L, L]^2$. We will consider a rotating velocity field with $u = -\omega y$ and $v = \omega x$. The initial condition is given by a Gaussian distribution:

$$C(\vec{x}, 0) = exp(-\sigma|\vec{x} - \vec{x}_c|^2).$$

The analytical solution to the differential equation with the initial condition is

$$C(x, y, t) = e^{-\sigma[(x\cos(\omega t)+y\sin(\omega t))^2+(y\cos(\omega t)-x\sin(\omega t)-\frac{3}{5})^2]}.$$

For simplicity, apply homogeneous essential boundary condition in the FEM implementation,

$$C(x, y) = 0, \quad \text{on } \partial\Omega.$$

Use

$$\omega = \frac{5\pi}{6},\ \vec{x}_c = (0, \frac{3}{5}),\ \sigma = \frac{125 \times 1000}{33^2},\ L = 1.$$

a) Develop a weak Galerkin formulation using a Crank-Nicolson time stepper for this problem.
b) Apply linear triangles, and formulate the mass matrix $M^{(n)}$, convection matrices $A^{(n)}$ and $B^{(n)}$. You can use analytical integrators.
c) Run irregular triangles at three levels of resolution. Evaluate the error for each case by comparing the numerical solution with the analytical solution at $t = 2.4$ (that is when the plume make one complete rotation). I suggest using a root mean square error.

Notes: Irregular grid for this domain at three levels of resolution will be sent to you - it will make this much easier for you. The irregmesh files contain the nodal coordinates and the irregtable files are the element connectivity tables. Mesh 1 is the coarsest and mesh 3 is the finest.